TRINITY COLLEGE

Year 11 Specialist Units 1,2
Test 6 2021

Calculator Free

Complex Numbers, Mathematical Induction

STUDENT'S NAME _____

DATE: Monday 20 September **TIME:** 50 minutes **MARKS**: 51

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, 1 A4 page of notes

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (4 marks)

If one root of a quadratic equation is -2 + 5i, determine the quadratic equation in the form $ax^2 + bx + c = 0$

2. (4 marks)

The sum of two number is -1 and the product of those numbers is 1. Determine the two numbers.

3. (6 marks)

Given $z = 5 + 2i$

(a) determine z^2 [2]

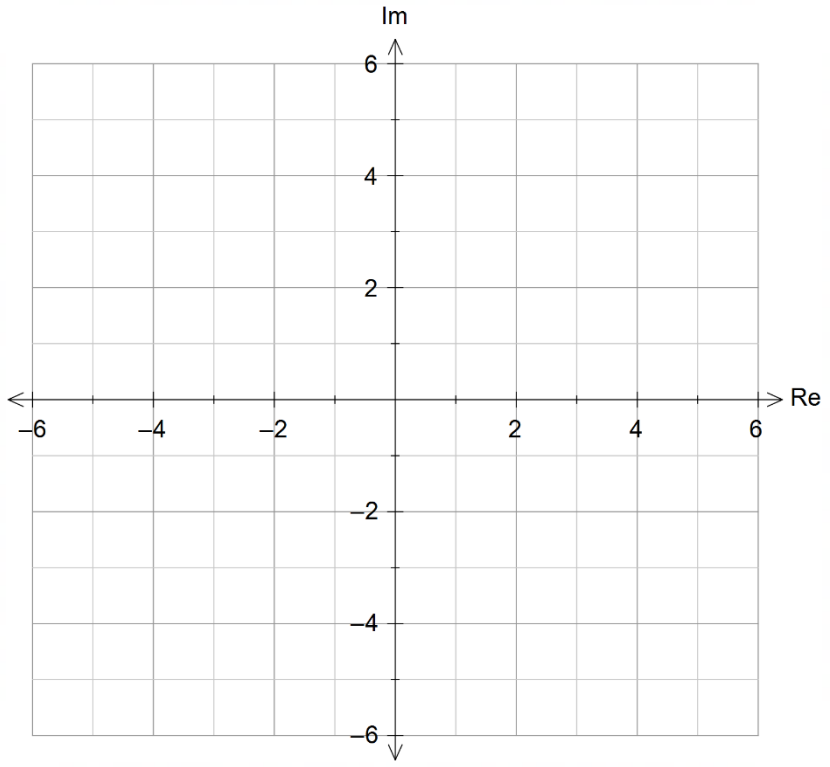
(b) determine $(\overline{z})^2$ [2]

(c) describe the relationship between z^2 and $(\overline{z})^2$ [2]

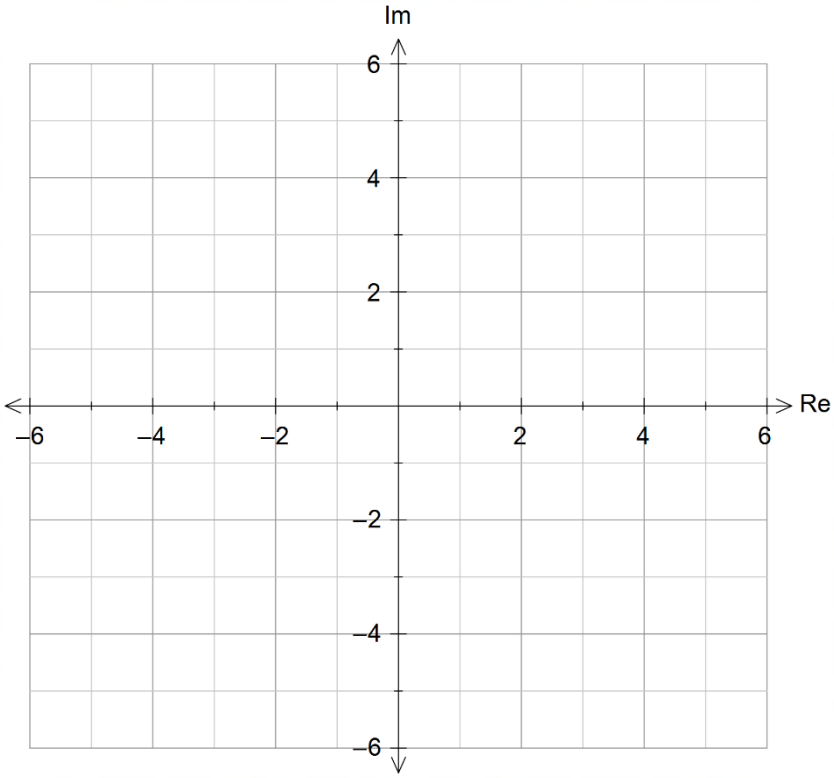
4. (5 marks)

For the complex number z, where $z = x + iy$

(a) Sketch $z - \overline{z} = 4i$ [3]



(b) Sketch $\text{Im}\, z > 2$ and $\text{Re}\, z \leq 2$ [2]



5. (8 marks)

Determine the complex number z, in the form $a+bi$, if

(a) $(z-2)^2+3=0$ [4]

(b) $2z+3=i(\overline{z})-5$ [4]

6. (9 marks)

Given $z = 2 - 5i$ and $w = 1 + 6i$, determine

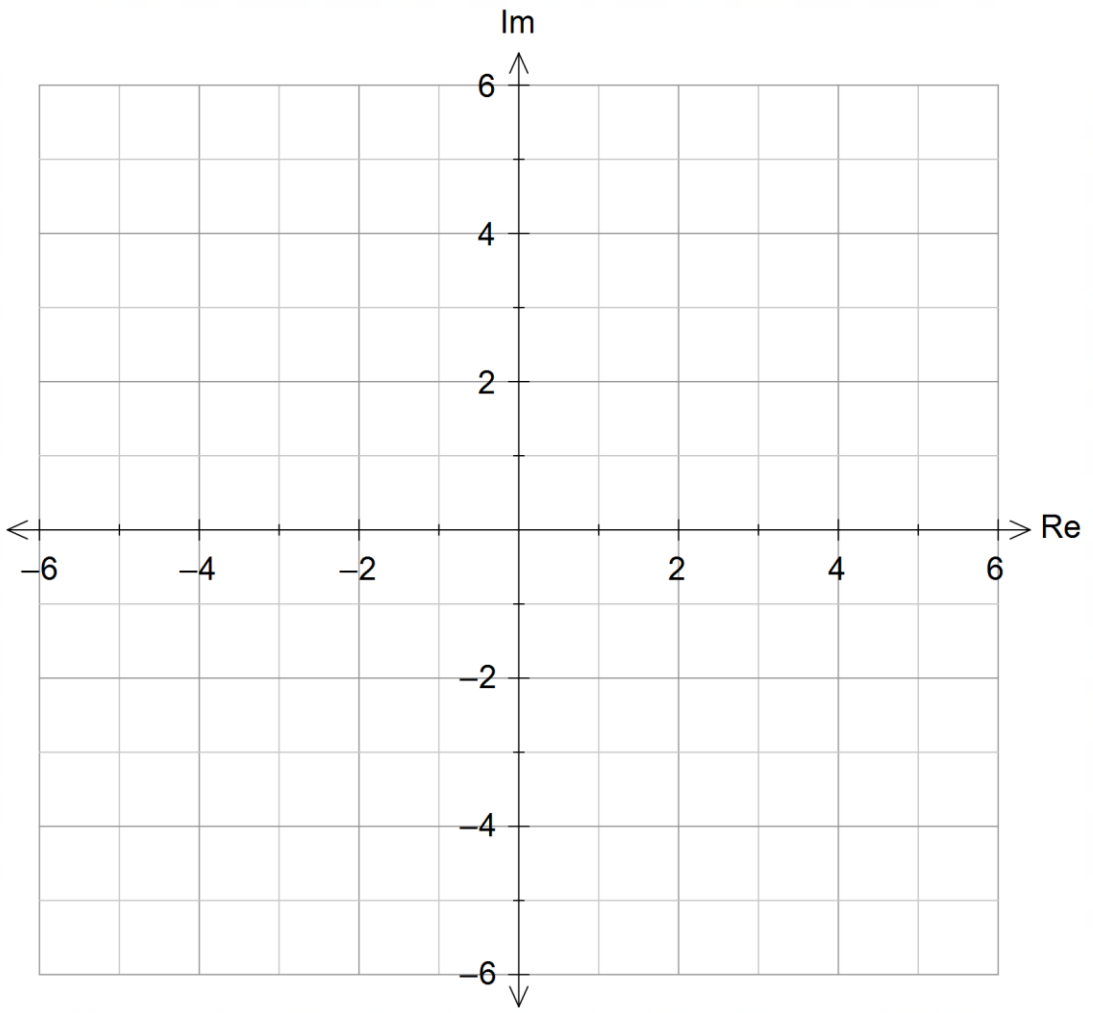
(a) $iz + \bar{w}$ [3]

(b) $\dfrac{i}{w}$ [3]

(c) $\text{Im}\left(\dfrac{z}{-i}\right)$ [3]

7. (8 marks)

If $w = -2 + 3i$, on the axes below plot the following.



(a) wi^3 [3]

(b) $|w|$ [2]

(c) $\dfrac{\overline{w}}{i^3}$ [3]

8. (7 marks)

Use mathematics induction to prove $n! > 2^n$ for n a positive integer greater than or equal to 4.